

The masses of the mesons and baryons.

Part I. The integer multiple rule

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From the well-known decays of the particles follows that the mesons and baryons consist of a γ -branch and a neutrino branch. From the well-known masses of the particles follows that the masses of the γ -branch particles are integer multiples of the mass of the π^0 meson, within 3%, in spite of differences in spin, isospin, strangeness and charm. The average factor in front of the integer multiples of $m(\pi^0)$ of the γ -branch particles is 1.0073 ± 0.0184 . The masses of the ν -branch particles are integer multiples of the mass of the π^\pm mesons, times a factor 0.86 ± 0.02 . The existence of the integer multiple rule can be verified from the Particle Physics Summary using a calculator.

The spectrum of the masses of the particles

The masses of the elementary particles are the best-known and most characteristic property of the particles. It seems to be important for the theoretical explanation of the masses of the particles to find a simple relationship between the different masses, as the formula for the Balmer series was important for the explanation of the spectrum of hydrogen. We will limit attention here to the mesons and baryons all of which are unstable, but for the proton. However, the lifetime of the mesons and baryons is so long as compared to the period of the basic frequency $\nu = mc^2/h$ that the mesons and baryons have often been categorized as “stable” or “elementary” particles. The masses of the so-called “stable” mesons and baryons are given in the “Particle Physics Summary” [1], and are reproduced with other data in Tables I,II.

It is obvious that any attempt to explain the masses of the elementary particles should begin with the particles that are affected by the fewest parameters. These are certainly the particles without isospin ($I = 0$) and without spin ($J = 0$), but also with strangeness $S = 0$, and charm $C = 0$. Looking at the particles with $I, J, S, C = 0$ it is startling to find that their masses are quite close to integer multiples of the mass of the π^0 meson. It is $m(\eta) = (1.0140 \pm 0.0003) \cdot 4m(\pi^0)$, and $m(\eta') = (1.0137 \pm 0.00015) \cdot 7m(\pi^0)$. We also note that the average mass ratios $[m(\eta)/m(\pi^0) + m(\eta)/m(\pi^+)]/2 = 3.9892 = 0.9973 \cdot 4$, and $[m(\eta')/m(\pi^0) + m(\eta')/m(\pi^+)]/2 = 6.9791 = 0.9970 \cdot 7$ are good approximations to the integers 4 and 7. Three particles seem hardly to be sufficient to establish a rule. However, if we look a little further we find that $m(\Lambda) = 1.0332 \cdot 8m(\pi^0)$ or $m(\Lambda) = 1.0190 \cdot 2m(\eta)$. We note that the Λ par-

ticle has spin $\frac{1}{2}$, not spin 0 as the π^0 , η , η' , mesons. Nevertheless, the mass of Λ is close to $8m(\pi^0)$. Furthermore we have $m(\Sigma^0) = 0.9817 \cdot 9m(\pi^0)$, $m(\Xi^0) = 0.9742 \cdot 10m(\pi^0)$, $m(\Omega^-) = 1.0325 \cdot 12m(\pi^0) = 1.0183 \cdot 3m(\eta)$, (Ω^- is charged and has spin $\frac{3}{2}$). Finally the masses of the charmed baryons are $m(\Lambda_c^+) = 0.9958 \cdot 17m(\pi^0) = 1.0232 \cdot 16m(\pi^+) = 1.024 \cdot 2m(\Lambda)$, $m(\Sigma_c^0) = 1.0093 \cdot 18m(\pi^0)$, $m(\Xi_c^0) = 1.0167 \cdot 18m(\pi^0)$, and $m(\Omega_c^0) = 1.0017 \cdot 20m(\pi^0)$. Now we seem to have enough material to formulate the integer multiple rule, according to which the masses of the η , η' , Λ , Σ^0 , Ξ^0 , Ω^- , Λ_c^+ , Σ_c^0 , Ξ_c^0 and Ω_c^0 particles are, in a first approximation, integer multiples of the mass of the π^0 meson, although some of the particles have spin, and may also have charge as well as strangeness and charm. A consequence of the integer multiple rule must be that the ratio of any meson or baryon mass divided by the mass of another meson or baryon is equal to the ratio of two integer numbers. And indeed, for example, $m(\eta)/m(\pi^0)$ is practically two times (exactly $0.9950 \cdot 2$) the ratio $m(\Lambda)/m(\eta)$, there is also the ratio $m(\Omega^-)/m(\Lambda) = 0.9993 \cdot \frac{3}{2} = 0.9993 \cdot 1.5$. We have furthermore the ratios $m(\Lambda)/m(\eta) = 1.019 \cdot 2$, $m(\Omega^-)/m(\eta) = 1.018 \cdot 3$, $m(\Lambda_c^+)/m(\Lambda) = 1.02399 \cdot 2$, $m(\Sigma_c^0)/m(\Sigma^0) = 1.0281 \cdot 2$, and $m(\Omega_c^0)/m(\Xi^0) = 1.0282 \cdot 2$.

We will call, for reasons to be explained later, the particles discussed above, which follow in a first approximation the integer multiple rule, the γ -branch of the particle spectrum. The mass ratios of these particles are listed in Table I. The deviation of the mass ratios from exact integer multiples of $m(\pi^0)$ is at most 3.3%, the average of the factors in front of the integer multiples of $m(\pi^0)$ of the ten γ -branch particles in Table I is 1.0073 ± 0.0184 . From a least square analysis follows that the masses of the eleven particles on Table I obey the formula $m = 1.0059N + 0.0074$ with a correlation coefficient

cient $r = 0.999$. The consequences of the combination of spin, isospin, strangeness and charm are difficult to assess. Nevertheless, even the combination of four different parameters does not change the integer multiple rule by more than 3.3%. To put this into perspective we note that the masses of the π^\pm mesons and the π^0 meson differ already by 3.4%.

Spin seems to have a profound effect on the mass of a particle. Changing the spin from zero for the π^0 , η , η' , mesons to spin $\frac{1}{2}$ for the Λ baryon is accompanied by a mass twice the mass of the η meson, it is $m(\Lambda) = 1.0190 \cdot 2m(\eta)$. The isospins of η and Λ are both zero. The change of S and the baryon number B which accompany the formation of Λ seems to have little effect on the mass of the Λ particle and the other baryons, as follows from the masses of the baryons whose S changes from -1 to -3 . We find it most interesting that spin $\frac{3}{2}$ is accompanied with a mass three times $m(\eta)$, the Ω^- particle; whereas spin $\frac{1}{2}$ is accompanied by a mass two times $m(\eta)$, the Λ particle.

Searching for what the π^0 , η , η' , Λ , Σ^0 , Ξ^0 , Ω^- particles have else in common, we find that the principal decays (decays with a fraction $> 1\%$) of these particles, as listed in Table I, involve primarily γ 's, the characteristic case is $\pi^0 \rightarrow \gamma\gamma$ (98.8%). The next most frequent decay product of the heavier particles of the γ -branch are π^0 mesons which again decay into $\gamma\gamma$. To describe the decays in another way, the principal decays of the particles listed above take place *always without the emission of neutrinos*; see Table I. There the decays and the fractions of the principal decay modes are listed, as they are given in the Particle Physics Summary. We cannot consider decays with fractions $< 1\%$. We will refer to the particles whose masses are approximately integer multiples of the mass of the π^0 meson, and which decay without the emission of neutrinos, as the γ -branch of the particle spectrum.

To summarize the facts concerning the γ -branch. Within about 3% the masses of the particles of the γ -branch are integer multiples (namely 4, 7, 8, 9, 10, 12, and even 17, 18, 20) of the mass of the π^0 meson. It is improbable that nine particles have masses so close to integer multiples of $m(\pi^0)$ if there is no correlation between them and the π^0 meson. It has, on the other hand, been argued that the integer multiple rule is a numerical coincidence. But the probability that the mass ratios of the γ -branch fall by coincidence on integer numbers between 1 and 20 instead on all possible percentage values between 1 and 20 is smaller than 10^{-20} , i.e., nonexistent. The integer multiple rule is not affected by more than 3% by the spin, the isospin, the strangeness, and by charm. The integer multiple rule seems even to apply to the Ω^- and Λ_c^+ particles, although they are charged. In order for the integer multiple rule to be valid the deviation of the ratio $m/m(\pi^0)$ from an integer number must be smaller than $1/2N$, where N is the integer number closest to the actual ratio $m/m(\pi^0)$. That means that the permissible deviation decreases rapidly with N . All particles of the γ -branch have deviations smaller than $1/2N$.

The remainder of the stable mesons and baryons are the π^\pm , $K^{\pm,0}$, p , n , $D^{\pm,0}$ and D_S^\pm particles. These are in gen-

eral charged, exempting the K^0 and D^0 mesons and the neutron n , in contrast to the particles of the γ -branch, which are in general neutral. It does not make a significant difference whether one considers the mass of a particular charged or neutral particle. After the π mesons, the largest mass difference between charged and neutral particles is that of the K mesons (0.81%), and thereafter all mass differences between charged and neutral particles are $< 0.5\%$. The integer multiple rule does not immediately apply to the masses of the charged particles if $m(\pi^\pm)$ (or $m(\pi^0)$) is used as reference, because $m(K^\pm) = 0.8843 \cdot 4m(\pi^\pm)$. $0.8843 \cdot 4$ is far from integer. Since the masses of the π^0 meson and the π^\pm meson differ by only 3.4% it has been argued that the π^\pm mesons are, but for the isospin, the same particles as the π^0 mesons, and that therefore the π^\pm cannot start another particle branch. However, this argument is not supported by the completely different decays of the π^0 mesons and the π^\pm mesons. The π^0 meson decays almost exclusively into $\gamma\gamma$ (98.8%), whereas the π^\pm mesons decay practically exclusively into μ -mesons and neutrinos, e.g. $\pi^+ \rightarrow \mu^+ + \nu_\mu$ (99.987%). Furthermore, the lifetimes of the π^0 and the π^\pm mesons differ by nine orders of magnitude, being $\tau(\pi^0) = 8.4 \cdot 10^{-17}$ sec *versus* $\tau(\pi^\pm) = 2.6 \cdot 10^{-8}$ sec.

If we make the π^\pm mesons the reference particles of the ν -branch, then we must multiply the mass ratios $m/m(\pi^\pm)$ of the above listed particles with a factor 0.861 ± 0.022 , as follows from the mass ratios listed on Table II. The integer multiple rule may, however, apply directly if one makes $m(K^\pm)$ the reference for masses larger than $m(K^\pm)$. The mass of the proton is $0.9503 \cdot 2m(K^\pm)$, which is only a fair approximation to an integer multiple. There are, on the other hand, outright integer multiples in $m(D^\pm) = 0.9961 \cdot 2m(p)$, and in $m(D_S^\pm) = 0.9968 \cdot 4m(K^\pm)$. We note that the spin $\frac{1}{2}$ of the proton is associated with a mass twice the mass of the spinless K meson, just as it was with the spin of the Λ particle, which is associated with a mass twice the mass of the spinless η meson. We note further that the spin of the D^\pm mesons, whose mass is nearly $2m(p)$, is zero, whereas the spin of the proton is $\frac{1}{2}$. It appears that the superposition of two particles of the same mass and with spin $\frac{1}{2}$ can cancel the spin. On the other hand, it appears that the superposition of two particles of equal mass without spin can create a particle with spin $\frac{1}{2}$.

Contrary to the particles of the γ -branch, the charged particles decay preferentially with the emission of neutrinos, the foremost example is $\pi^+ \rightarrow \mu^+ + \nu_\mu$ with a fraction of 99.987%. Neutrinos characterize the weak interaction. We will refer to the charged particles listed in Table II as the *neutrino branch* (ν -branch) of the particle spectrum. We emphasize that a weak decay of the particles of the ν -branch is by no means guaranteed, the proton is stable and there are decays as, e.g., $K^+ \rightarrow \pi^+ \pi^- \pi^+$ (5.59%), but the subsequent decays of the π^\pm mesons lead, on the other hand, to neutrinos and e^\pm . There is also the K^0 particle, which poses a problem because the principal primary decays of K_S^0 take place without the emission of neutrinos, but many of the secondary decays emit neutrinos. On the other hand, 2/3 of the primary decays of K_L^0 emit neutrinos. For comparison 63.6% of K^+ decay into $\mu^+ + \nu_\mu$. The

Table 1: The γ -branch of the particle spectrum.

	$m/m(\pi^0)$	multiples	decay	fraction (%)	spin
π^0	1	$1 \cdot \pi^0$	$\gamma\gamma$ $e^+e^-\gamma$	98.798 1.198	0
η	4.0559	$1.0140 \cdot 4\pi^0$	$\gamma\gamma$ $3\pi^0$ $\pi^+\pi^-\pi^0$ $\pi^+\pi^-\gamma$	39.25 32.1 23.1 4.78	0
η'	7.096	$1.0137 \cdot 7\pi^0$	$\pi^+\pi^-\eta$ $\rho^0\gamma$ $\pi^0\pi^0\eta$ $\omega\gamma$ $\gamma\gamma$	43.7 30.2 20.8 3.02 2.12	0
Λ	8.26577	$1.0332 \cdot 8\pi^0$ $1.0190 \cdot 2\eta$	$p\pi^-$ $n\pi^0$	63.9 35.8	$\frac{1}{2}$
Σ^0	8.8352	$0.9817 \cdot 9\pi^0$	$\Lambda\gamma$	100	$\frac{1}{2}$
Ξ^0	9.7417	$0.9742 \cdot 10\pi^0$	$\Lambda\pi^0$	99.54	$\frac{1}{2}$
Ω^-	12.390	$1.0326 \cdot 12\pi^0$ $0.9986 \cdot 12\pi^-$ $1.0183 \cdot 3\eta$	ΛK^- $\Xi^0\pi^-$ $\Xi^-\pi^0$	67.8 23.6 8.6	$\frac{3}{2}$
Λ_c^+	16.928	$0.9958 \cdot 17\pi^0$ $0.9630 \cdot 17\pi^+$	many		$\frac{1}{2}$
Σ_c^0	18.167	$1.0093 \cdot 18\pi^0$	$\Lambda_c^+\pi^-$	≈ 100	$\frac{1}{2}$
Ξ_c^0	18.302	$1.0167 \cdot 18\pi^0$			$\frac{1}{2}$
Ω_c^0	20.033	$1.0017 \cdot 20\pi^0$			$\frac{1}{2}$

decays of the particles in the ν -branch follow a mixed rule, either weak or electromagnetic.

To summarize the facts concerning the ν -branch of the mesons and baryons. The masses of these particles seem to follow an integer multiple rule if one uses the K^\pm meson as reference, however the masses are not integer multiples of the π^\pm mesons but share a common factor 0.86 ± 0.02 .

We do not want to discuss here the bottom mesons, and the bottom baryon. It is easy to associate the masses of the bottom mesons with integer multiples of the π^0 meson. For example, $m(B^0) = 1.0029 \cdot 39m(\pi^0)$ and $m(B_S^0) = 0.9945 \cdot 40m(\pi^0)$ or $1.0199 \cdot 39m(\pi^0)$. The latter numbers show the difficulty in identifying the proper multiple. In this case the difference of the integer multiples is one out of forty or 2.5%, which is too small in order to make a proper identification of the multiple. There are, however, the $c\bar{c}$ mesons η_c and χ_{c0} , with $I, J = 0$, whose masses are $1.0035 \cdot 22m(\pi^0)$ and $1.0120 \cdot 25m(\pi^0)$, which therefore fall into the range of the masses of the γ -branch considered here, and which are good approximations to integer multiples of $m(\pi^0)$.

Table 2: The ν -branch of the particle spectrum

	$m/m(\pi^+)$	multiples	decay ¹	fraction (%)	spin
π^\pm	1	$1 \cdot \pi^+$	$\mu^+\nu_\mu$	99.987	0
K^\pm	3.53713	$0.8843 \cdot 4\pi^+$	$\mu^+\nu_\mu$ $\pi^+\pi^0$ $\pi^+\pi^-\pi^+$ $\pi^0\mu^+\nu_\mu$ $\pi^0e^+\nu_e$	63.57 21.16 5.59 3.18 4.82	0
p^+	6.722595	$0.8403 \cdot 8\pi^+$ $0.9503 \cdot 2K^+$ $0.9604 \cdot 7\pi^+$	stable		$\frac{1}{2}$
D^\pm	13.393	$0.8370 \cdot 16\pi^+$ $0.9466 \cdot 4K^+$ $0.9961 \cdot 2p$	$e^+ anything$ $K^- anything$ $\bar{K}^0 anything$ $+ K^0 anything$ $\eta anything$	17.2 24.2 59 <13	0
D_S^\pm	14.104	$0.8815 \cdot 16\pi^+$ $0.9968 \cdot 4K^+$	$K^- anything$ $\bar{K}^0 anything$ $+ K^0 anything$ $K^+ anything$ $e^+ anything$	13 39 20 <20	0

¹ The particles with negative charge have conjugate charges of the decays listed.

2 Summary

In spite of differences in charge, spin, strangeness, and charm the masses of the stable mesons and baryons of the γ -branch are, within at most 3.3%, integer multiples of the mass of the π^0 meson. Correspondingly, the masses of the particles of the ν -branch are, after multiplication with a factor 0.86 ± 0.02 , integer multiples of the mass of the π^\pm mesons. The validity of the integer multiple rule can easily be verified from the Particle Physics Summary using a calculator. The integer multiple rule suggests that the particles are the result of superpositions of modes and higher modes of a wave equation. Such a theory will be presented in a following paper.

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REFERENCE

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